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PERTURBATION-WAVE PROPAGATION IN PETROLEUM CONTAINING TAR

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A study is made on nonlinear-wave propagation in petroleum containing tar.

Recent studies have shown that high-viscosity petroleum containing much tar shows relaxation behavior [1, 2], which is due to clusters consisting of hundreds or more macromolecules. Such a medium resembles a conglomerate material [3] in having local deformation viscosity due to the compressibility and the elasticity of these particles, which leads to pressure relaxation. In the propagation of a nonstationary wave in such a medium, there may be an effect from the spread in the impact momentum, as has been found in experiments [4].

The following is a system of equations describing the planar one-dimensional motion of such a medium, which includes the equations of continuity and motion together the Tait equation for each phase [5]

$$\frac{\partial\rho_1}{\partial t}+\frac{\partial\rho_1 v_1}{\partial x}=0,\ \frac{\partial\rho_2}{\partial t}+\frac{\partial\rho_2 v_2}{\partial x}=0,$$

$$\rho_1=\rho_1^0\alpha_1,\ \rho_2=\rho_2^0\alpha_2,\ \alpha_1+\alpha_2=1,\ \rho_1\frac{d_1v_1}{dt}=-\alpha_1\frac{\partial p_1}{\partial x}-F_\mu,$$

$$\rho_2\frac{d_2v_2}{dt}=-\alpha_2\frac{\partial p_1}{\partial x}+F_\mu,\ F_\mu=\alpha_{01}\alpha_{02}K_\mu(v_2-v_1), \tag{1}$$

$$p_i=\frac{\rho_{0i}^0c_i^2}{n_i}\left[\left(\frac{\rho_i^0}{\rho_{0i}^0}\right)^{n_i}-1\right],\ \frac{d_i}{dt}=\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x},\ i=1,\ 2.$$

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System (1) is supplemented with the relaxation equation relating the pressures of the phases, which incorporates the inertial character of the pressure recovery on loading:

$$p_1 = p_2 + \Theta \frac{\partial p_2}{\partial t}, \tag{2}$$

where Θ is the parameter characterizing the relaxation behavior.

We consider perturbations of small but finite amplitude with characteristic frequencies $\omega \ll K_\mu, 1/\Theta$; then from (1) and (2) we have as follows up to terms of order ε^2 ($\varepsilon \sim v_i/c_0 \sim p_i'/\rho_0 c_0^2 \sim \alpha_i'/\alpha_{0i}$):

$$\frac{1}{c_0^2}\frac{\partial^2 U}{\partial t^2} - \frac{\partial^2 V}{\partial x^2} = A\frac{\partial^2 V^2}{\partial t^2} + B\frac{\partial^2 V^2}{\partial x \partial t} + \frac{1}{c_0}\frac{\partial^2 V^2}{\partial x^2} - \frac{\alpha_{02}\Theta\beta_2}{\beta c_0}\frac{\partial^3 V}{\partial x \partial t^2}, \tag{3}$$

$$U - V = \frac{\rho_{01}^0 \rho_{02}^0}{\rho_0 K_\mu}\frac{\partial}{\partial t}\left(V - \frac{\rho_0}{\rho_\infty}U\right), \tag{4}$$

$$A = -\frac{\alpha_{01}\rho_{01}^0\beta_1 + \alpha_{02}\rho_{02}^0\beta_2}{2c_0} + \frac{\alpha_{01}\alpha_{02}(\beta_1 - \beta_2)(\rho_{01}^0 - \rho_{02}^0)}{2c_0},$$

$$B = -\frac{1}{2c_0^2} - \frac{\alpha_{02}(n_2 - 1)\beta_2^2 + \alpha_{01}(n_1 - 1)\beta_1}{2\beta^2 c_0^2} + \frac{\alpha_{01}\alpha_{02}(\beta_1 - \beta_2)^2}{\beta^2 c_0^2}.$$

Here primes denote the deviations of the corresponding quantities from their equilibrium values.

System (3) and (4) describes the motion of weak waves in saturated porous media, which has been examined in detail in [6], if one uses the linear approximation and neglects pressure relaxation between phases. If we assume that the dissipative and nonlinear terms are small by comparison with the others, they can be considered as a certain perturbation. The effects of these terms are small only at distances of the order of the wavelength, but their contributions accumulate at large distances and become substantial. We therefore apply the slowly-varying amplitude method [7] to (3) and (4) to get the corresponding coordinate system $\tau = t - x/c_0$ that

$$\frac{\partial V}{\partial x} - \alpha V\frac{\partial V}{\partial \tau} = \gamma\frac{\partial^2 V}{\partial \tau^2}, \tag{5}$$

where

$$\alpha = \frac{3}{2c_0^2} + \frac{\alpha_{01}\alpha_{02}(\beta_1 - \beta_2)^2}{\beta^2 c_0^2} + \frac{\alpha_{01}\alpha_{02}(\beta_1 - \beta_2)(\rho_{01}^0 - \rho_{02}^0)}{2} - \frac{\alpha_{01}\rho_{01}^0\beta_1 + \alpha_{02}\rho_{02}^0\beta_2}{2} + \frac{\alpha_{02}(n_2 - 1)\beta_2^2 + \alpha_{01}(n_1 - 1)\beta_1^2}{2\beta^2 c_0^2},$$

$$\gamma = \frac{\rho_{01}^0 \rho_{02}^0}{2\rho_0 c_0 K_\mu}\left(\frac{\rho_0}{\rho_\infty} - 1\right) + \frac{\alpha_{02}\beta_2\Theta}{2\beta c_0}.$$

In deriving (5), we have used the fact that the frequency is small by comparison with the reciprocal of the relaxation time, i.e., $\omega\Theta \ll 1$; this condition should be obeyed for the upper limit to the wave spectrum if the signal is not monochromatic.

Equation (5) is a Burgers equation, and its solutions are familiar [8]; the dissipative term on the right in (5) is due to differences in phase inertia and to the relaxation behavior, being dependent on the volume concentrations of the phases, the densities, the coefficient of friction between phases, and the relaxation time.

The experimental data of [4] show that the main contribution to the dissipative term in (5) comes from the inertial pressure recovery in the mixture, while the contribution due to differences in inertia between phases is small. This enables one to estimate the characteristic pressure recovery time. The width Γ of the front of a stationary shock wave is $\Gamma = \gamma\omega/u_0\alpha$, where u_0 is the amplitude of the initial perturbation. One can use the waveforms for shock-wave propagation in tar-bearing oils given in [4] ($u_0 \simeq 10$ m/sec, $\alpha_{02} \simeq 0.1$, $\beta_1 \simeq 0.58\cdot10^{-9}$ N·sec/kg, $\beta_2 \simeq 0.41\cdot10^{-9}$ N·sec/kg, $\rho_{01}^0 = 0.8\cdot10^3$ kg/m³, $\rho_{02}^0 = 10^3$ kg/m³, $\omega \simeq 1$ kHz and $\Gamma \simeq 1$) to get that $\Theta \simeq 10^{-4}$ sec.

Then the characteristic pressure-recovery time on loading is 10^{-4} sec, which corresponds to a shock-wave thickness of 0.1 m, which is somewhat less than the length of the relaxation zone in a bubble liquid [9].

The speed of sound is dependent on the volume concentrations of the phases, the compressibilities, and the densities; for the values given for ρ_i and β_i, i = 1, 2, the dependence of the speed of sound on concentration is in qualitative agreement with the measurement of [10] (the speed decreases as α_{02} increases).

These relationships enables one to use a small number of shock-wave propagation recordings for a known tar concentration to evaluate the relaxation in the phase interaction.

The analysis thus shows that the wave propagation is affected from two-velocity effects arising from the relative motion of the continuous phase (petroleum) and dispersed phase (tar) only to a small extent by comparison with the effects from the pressure-recovery relaxation.

NOTATION

v_i, p_i, ρ_i, α_i, velocity, pressure, true density, and volume concentration of phases, i = 1 corresponds to the carrier phase, i = 2 to the dispersed phase; K_μ, interphase friction coefficient; c_i, equilibrium velocity of sound in phase; n_i, constants; ρ^0_{0i}, α_{0i}, equilibrium values of phase densities and volume concentrations, respectively; $c_0 = (\beta\rho_0)^{-1/2}$, equilibrium velocity of sound in the medium; $\rho_0 = \alpha_{01}\rho^0_{01} + \alpha_{02}\rho^0_{02}$, equilibrium density of the medium; $\beta_i = (\rho^0_{0i}c^2{}_i)^{-1/2}$, compressibility of the i-th phase; $V = \alpha_{01}v_1 + \alpha_{02}v_2$, mean volume velocity of the medium; $U = \rho_0{}^{-1}(\alpha_{01}\rho^0_{01}v_1 + \alpha_{02}\rho^0_{02}v_2)$, mean mass velocity of the medium; $\beta = \alpha_{01}\beta_1 + \alpha_{02}\beta_2$, compressibility; $\rho_\infty{}^{-1} = \alpha_{01}\rho^{-1}_{01} + \alpha_{02}\rho^{-1}_{02}$.

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